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Effect of the smaller mass-squared difference for the long base-line neutrino experiments

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Abstract

Usually, neutrino oscillation experiments are analyzed within the two-flavor framework which is governed by 1 mass-squared difference and 1 mixing angle. But there are 6 parameters, 2 mass-squared differences, 3 mixing angles, and 1 CP phase within the three-flavor framework. In this article, we estimate the effect from the smaller mass-squared difference, the other mixing angles, and the CP phase, which we call three-flavor effect, for the determination of the mass-squared difference and the mixing angle from the ν_μ's survival and transition probability with the two-flavor analysis. It is found that the mass-squared difference from the two-flavor analysis is slightly shifted from the larger mass-squared difference by the three-flavor effect. The order of magnitude of the three-flavor effect for the mass-squared difference is comparable with that of the expected error for the mass-squared difference of the two-flavor analysis in the future long base-line neutrino oscillation experiments. The CP phase dependence of the $\nu_\mu \to \nu_e$ transition probability is also shown.

1 introduction

The three neutrino framework has 9 physical parameters: 3 neutrino masses, 3 mixing angles, and 3 CP violating phase, if neutrinos are Majorana particles. Neutrino oscillation experiments are sensitive to 6 parameters: 2 mass-squared difference, 3 mixing angles, and 1 CP phase. Usually, data from the experiments are analyzed within the two-flavor framework, which is governed by only 1 mass-squared difference and 1 mixing angle. So far, for the long base-line neutrino oscillation experiments, we have been able to neglect the effect from the smaller mass-squared difference, the other mixing angles, and CP phase, which

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we call three-flavor effect. This is because the error of the larger mass-squared difference, related to the atmospheric neutrino observations [1] and the on-going long base-line experiment K2K [2], is larger than the smaller mass-squared difference, which is obtained from solar neutrino observations [3] and the reactor neutrino experiment KamLAND [4].

The long base-line neutrino oscillation experiments in future [5, 6, 7] plan to measure the mass-squared difference and mixing parameter precisely. Because the order of magnitude of the ratio between the smaller mass-squared difference and the larger one is supposed to be similar to that of expected error of the future long base-line experiments, it is necessary to take into account the contribution of the three-flavor effect in the determination of the mass-squared difference and mixing angle from the future long base-line precision measurements. In this article, we estimate this effect using the three-flavor framework. We discard the result of the LSND experiment [8]. Obviously, this analysis is, in general, not valid, if the LSND result is confirmed by the MiniBooNE experiment [9]. From the survival probability of ν_μ, we find that the larger mass-squared difference is shifted by the three-flavor effect and that the order of magnitude of this shift depends on the neutrino energy and is similar to that of the smaller mass-squared difference. We obtain the same result from the transition probability $\nu_\mu \to \nu_e$, and also find the CP phase dependence for the transition probability. A lot of groups have analyzed the experimental data with the three-flavor framework numerically [10]. However, these analyses cannot point out the specific reason for the value of parameters. In this paper, we point out the specific contribution to the parameters from the three-flavor effect. We think that these formulations are useful to study in the numerical analysis.

This article is organized as follows. In section 2, we will show the useful notations and the convenient form of the probability for easy estimating the contribution of the three-flavor effect. In section 3, we will estimate the three-flavor effect for the ν_μ disappearance mode. We will also estimate the three-flavor effect for the $\nu_\mu \to \nu_e$ transition mode, in section 4. Finally, we will be devoted to the summary in the last section.

2 notations

In the three neutrino framework, and in the basis in which the charged leptons are diagonal, the three weak interaction eigenstates, ν_α $(\alpha = e, \mu, \tau)$ are expressed as

$$\nu_\alpha = \sum_{i=1}^{3} (V_{\mathrm{MNS}})_{\alpha i} \; \nu_i \, , \tag{1}$$

where ν_i are the three mass eigenstates and V_{MNS} is the Maki-Nakagawa-Sakata (MNS) matrix [11]. We adopt the following parameterization [12]

$$V_{\mathrm{MNS}} = U\mathcal{P} = U \;\, \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1) \, , \tag{2}$$

where $\mathcal{P}$ cannot be determined from the neutrino oscillation experiment. The matrix U, which has three mixing angles and one phase, can be parameterized in the same way as the Cabibbo-Kobayashi-Maskawa matrix [13]. Because the present neutrino oscillation experiments constrain directly the elements U_{e2}, U_{e3}, and $U_{\mu 3}$, we find it most convenient to adopt the parameterization [14, 15], where these three elements in the upper-right corner of the U matrix are the independent parameters. Without losing generality, we can take U_{e2} and $U_{\mu 3}$ to be real and non-negative. By allowing U_{e3} to have the complex phase,

$$U_{e2}\,, U_{\mu 3} \geq 0, \quad U_{e3} \equiv |U_{e3}|\, e^{-i\phi} \;\; (0 \leq \phi < 2\pi)\,, \tag{3}$$

these $U_{e2}, U_{\mu 3}, |U_{e3}|$, and ϕ are the four independent parameters. All the other elements of U are then determined by the unitary conditions,

$$U_{e1} = \sqrt{1 - |U_{e3}|^2 - |U_{e2}|^2}\,, \qquad U_{\tau 3} = \sqrt{1 - |U_{e3}|^2 - |U_{\mu 3}|^2}\,, \tag{4a}$$

$$U_{\mu 1} = -\frac{U_{e2}U_{\tau 3} + U_{\mu 3}U_{e1}U_{e3}^*}{1 - |U_{e3}|^2}\,, \quad U_{\mu 2} = \frac{U_{e1}U_{\tau 3} - U_{\mu 3}U_{e2}U_{e3}^*}{1 - |U_{e3}|^2}\,, \tag{4b}$$

$$U_{\tau 1} = \frac{U_{e2}U_{\mu 3} - U_{\tau 3}U_{e1}U_{e3}^*}{1 - |U_{e3}|^2}\,, \qquad U_{\tau 2} = -\frac{U_{\mu 3}U_{e1} + U_{e2}U_{\tau 3}U_{e3}^*}{1 - |U_{e3}|^2}\,. \tag{4c}$$

For the convenience, the independent parameters in the MNS matrix are rewritten as

$$U_{e3} \equiv \sin\theta_{13}\,, \qquad U_{e2} \equiv \sin\theta_{12}\cos\theta_{13}\,, \qquad U_{\mu 3} \equiv \sin\theta_{23}\cos\theta_{13}\,. \tag{5}$$

The atmospheric neutrino oscillation experiments, which measure the ν_μ survival probability determine the absolute values of the larger mass-squared differences and one-mixing angle [1] as

$$1.5 \times 10^{-3} < \left|\delta m^2_{\rm atm}\right| < 3.4 \pm 0.5 \times 10^{-3}{\rm eV}^2\,, \qquad \text{and} \qquad \sin^2 2\theta_{\rm atm} > 0.92\,, \tag{6}$$

at the 90% C.L. The K2K experiment [2] confirms the above results. These values are planed to measure more precisely, a few percent order by the future long base-line experiments [5, 6, 7]. The solar neutrino experiments, which measure the ν_e survival probability in the sun [3], and the KamLAND experiment which measure the $\bar{\nu}_e$ survival probability from the reactors [4], determine the smaller mass-squared difference and another mixing angle as

$$\delta m^2_{\rm sol} = 8.2^{+0.6}_{-0.5} \times 10^{-5}{\rm eV}^2\,, \qquad \text{and} \qquad \tan^2\theta_{\rm sol} = 0.40^{+0.09}_{-0.07}\,. \tag{7}$$

It is remarkable point that the order of the smaller mass squared-difference is as same as that of expected error of the future long base-line experiments [5, 6, 7]. Thus, it is necessary to take into account the contribution of the three-flavor effect in the determination of the mass-squared difference and mixing angle analytically, because experimental data are analyzed ordinary in the two-flavor framework. The CHOOZ reactor experiment [16] gives

the upper bound of the third mixing angle as

$$\begin{aligned}
\sin^2 2\theta_{\rm rct} &< 0.20 \quad \text{for} \quad \delta m^2 = 2.0\times 10^{-3}{\rm eV}^2\,,\\
\sin^2 2\theta_{\rm rct} &< 0.16 \quad \text{for} \quad \delta m^2 = 2.5\times 10^{-3}{\rm eV}^2\,,\\
\sin^2 2\theta_{\rm rct} &< 0.14 \quad \text{for} \quad \delta m^2 = 3.0\times 10^{-3}{\rm eV}^2\,,
\end{aligned} \tag{8}$$

at the 90% C.L. Since we can always take $|\Delta_{12}| < |\Delta_{13}|$ without loosing generality, we assume that Δ_{12} is from the results of solar neutrino and reactor anti-neutrino observations and Δ_{13} is related to the atmospheric and long base-line neutrino experiments. The sign of Δ_{12} is determined form the solar neutrino observation, $\Delta_{12} > 0$. However, that of Δ_{13} cannot be determined by any observations. In this article, we call $\Delta_{13} > 0$ "normal hierarchy" and $\Delta_{13} < 0$ "inverted hierarchy". Under the $|\Delta_{12}| < |\Delta_{13}|$ relation, $\theta_{\rm atm}$, $\theta_{\rm sol}$, and $\theta_{\rm rct}$ are related to the MNS matrix elements as :

$$\begin{aligned}
2U_{\mu 3}^2 &= 1 - \sqrt{1-\sin^2 2\theta_{\rm atm}}\,,\\
2\left|U_{e3}\right|^2 &= 1 - \sqrt{1-\sin^2 2\theta_{\rm rct}}\,,\\
2U_{e2}^2 &= 1 - |U_{e3}|^2 - \sqrt{(1-|U_{e3}|^2)^2 - \sin^2 2\theta_{\rm sol}}\,.
\end{aligned} \tag{9}$$

Starting from the flavor eigenstate α, the probability of finding the flavor eigenstate β at the base-line length L is, in vacuum,

$$P_{\nu_\alpha\to\nu_\beta} = \left|\sum_{j=1}^{3}(V_{\rm MNS})_{\beta j}\exp\left(-i\frac{m_j^2}{2E_\nu}L\right)(V_{\rm MNS})^*_{\alpha j}\right|^2 \tag{10a}$$

$$\begin{aligned}
&= \delta_{\alpha\beta} - 4{\rm Re}\left\{U_{\alpha 3}U^*_{\beta 3}U_{\beta 1}U^*_{\alpha 1} + U_{\alpha 2}U^*_{\beta 2}U_{\beta 3}U^*_{\alpha 3}\right\}\sin^2\frac{\Delta_{13}}{2}\\
&\quad - 4{\rm Re}\left\{U_{\alpha 2}U^*_{\beta 2}U_{\beta 1}U^*_{\alpha 1} + U_{\alpha 2}U^*_{\beta 2}U_{\beta 3}U^*_{\alpha 3}\right\}\sin^2\frac{\Delta_{12}}{2}\\
&\quad + 2{\rm Re}\left(U_{\alpha 2}U^*_{\beta 2}U_{\beta 3}U^*_{\alpha 3}\right)\left(\sin\Delta_{12}\sin\Delta_{13} + 4\sin^2\frac{\Delta_{12}}{2}\sin^2\frac{\Delta_{13}}{2}\right)\\
&\quad - 4J^{(\alpha,\beta)}_{\rm MNS}\left(\sin^2\frac{\Delta_{13}}{2}\sin\Delta_{12} - \sin^2\frac{\Delta_{12}}{2}\sin\Delta_{13}\right)\,,
\end{aligned} \tag{10b}$$

where $J^{(\alpha,\beta)}_{\rm MNS}$ is the Jarlskog parameter [17]:

$$\begin{aligned}
J^{(\alpha,\beta)}_{\rm MNS} &\equiv {\rm Im}\left((V_{\rm MNS})_{\alpha i}(V_{\rm MNS})^*_{\beta i}(V_{\rm MNS})_{\beta j}(V_{\rm MNS})^*_{\alpha j}\right)\\
&= {\rm Im}\left(U_{\alpha i}U^*_{\beta i}U_{\beta j}U^*_{\alpha j}\right) = -\frac{U_{e1}U_{e2}U_{\mu 3}U_{\tau 3}}{1-\left|U_{e3}\right|^2}{\rm Im}\left(U_{e3}\right) \equiv A\sin\phi\,,
\end{aligned} \tag{11}$$

which is defined to be positive for $(\alpha, \beta) = (e, \mu), (\mu, \tau), (\tau, e)$ and $(i, j) = (1, 2), (2, 3), (3, 1)$. A is the absolute value of the Jarlskog parameter. In addition, Δ_{ij} is

$$\Delta_{ij} \equiv \frac{m_j^2 - m_i^2}{2E_\nu} L = \frac{\delta m_{ij}^2}{2E_\nu} L \simeq 2.534 \frac{\delta m_{ij}^2 (\mathrm{eV}^2)}{E_\nu (\mathrm{GeV})} L(\mathrm{km}) \,, \tag{12}$$

where E_ν is the neutrino energy.

We rewrite eq. (10) in a form convenient to estimate the contribution of the smaller mass-squared difference,

$$P_{\nu_\alpha \to \nu_\beta} \equiv P_0(\alpha, \beta) + P_1(\alpha, \beta) \times \sin \Delta_{12} + P_2(\alpha, \beta) \times 4 \sin^2 \frac{\Delta_{12}}{2} \,, \tag{13}$$

where each term is

$$P_0(\alpha, \beta) = \delta_{\alpha\beta} - 4\mathrm{Re} \left\{ U_{\alpha 3} U_{\beta 3}^* U_{\beta 1} U_{\alpha 1}^* + U_{\alpha 2} U_{\beta 2}^* U_{\beta 3} U_{\alpha 3}^* \right\} \sin^2 \frac{\Delta_{13}}{2} \,, \tag{14a}$$

$$P_1(\alpha, \beta) = 2\mathrm{Re} \left(U_{\alpha 2} U_{\beta 2}^* U_{\beta 3} U_{\alpha 3}^* \right) \sin \Delta_{13} - 4 J_{\mathrm{MNS}}^{(\alpha,\beta)} \sin^2 \frac{\Delta_{13}}{2} \,, \tag{14b}$$

$$P_2(\alpha, \beta) = -\mathrm{Re} \left(U_{\alpha 1} U_{\beta 1}^* U_{\beta 2} U_{\alpha 2}^* + U_{\alpha 2} U_{\beta 2}^* U_{\beta 3} U_{\alpha 3}^* \cos \Delta_{13} \right) + J_{\mathrm{MNS}}^{(\alpha,\beta)} \sin \Delta_{13} \,. \tag{14c}$$

All the above formulas remain valid when replacing the mass-squared differences and the MNS matrix elements with the "effective" ones,

$$\Delta_{ij} \to \widetilde{\Delta}_{ij} \,, \quad U_{\alpha i} \to \widetilde{U}_{\alpha i} \,, \quad J_{\mathrm{MNS}}^{(\alpha,\beta)} \to \widetilde{J}_{\mathrm{MNS}}^{(\alpha,\beta)} \,, \tag{15}$$

as long as the matter density remains the same along the base-line. The effective parameters $\widetilde{U}_{\alpha i}$ are defined from the following Hamiltonian

$$\widetilde{U} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \widetilde{U}^\dagger = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \delta m_{12}^2 & 0 \\ 0 & 0 & \delta m_{13}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \,, \tag{16}$$

and $\widetilde{\Delta}_{ij}$ is defined as

$$\widetilde{\Delta}_{ij} \equiv \frac{\delta \widetilde{m}_{ij}^2}{2E_\nu} L = \frac{\lambda_j - \lambda_i}{2E_\nu} L \,. \tag{17}$$

The term a in eq. (16) stands for the matter effect [18],

$$a(E_\nu) = 2\sqrt{2} G_F n_e E_\nu = 7.56 \times 10^{-5} (\mathrm{eV}^2) \left(\frac{\rho}{\mathrm{g/cm}^3} \right) \left(\frac{E_\nu}{\mathrm{GeV}} \right) \,, \tag{18}$$

where n_e is the electron density of the matter, G_F is the Fermi constant, and ρ is the matter density. When $\delta m^2_{12} < a < \delta m^2_{13}$ and $U_{e3} \ll O(1)$, the effective mass-squared differences are written as

$$\widetilde{\Delta}_{13} \simeq \Delta_{13} - \Delta_{12}\cos^2\theta_{12}\,, \qquad \widetilde{\Delta}_{12} \simeq \frac{a}{2E}L - \Delta_{12}\cos 2\theta_{12}\,, \tag{19}$$

and the mixing angles become

$$\widetilde{\theta}_{23} \simeq \theta_{23}\,, \qquad \widetilde{\theta}_{13} \simeq \left(1 + \frac{a}{\delta m^2_{13}}\right)\theta_{13}\,, \qquad \tan 2\widetilde{\theta}_{12} \simeq \frac{\delta m^2_{12}\sin 2\theta_{12}}{\delta m^2_{12}\cos 2\theta_{12} - a}\,. \tag{20}$$

Hereafter, we use $U_{\alpha i}$ and Δ_{ij} instead of $\widetilde{U}_{\alpha i}$ and $\widetilde{\Delta}_{ij}$, because of simplicity. But we have to keep in mind that these values depend on the neutrino energy.

3 survival probability of ν_μ

From eqs. (13) and (14), the survival probability of ν_μ is written as

$$P_{\nu_\mu\to\nu_\mu} = P_0(\mu,\mu) + P_1(\mu,\mu)\times\sin\Delta_{12} + P_2(\mu,\mu)\times 4\sin^2\frac{\Delta_{12}}{2}\,, \tag{21}$$

where

$$P_0(\mu,\mu) = 1 - 4\left|U_{\mu 3}\right|^2\left(1 - \left|U_{\mu 3}\right|^2\right)\sin^2\frac{\Delta_{13}}{2}\,, \tag{22a}$$

$$P_1(\mu,\mu) = 2\left|U_{\mu 2}\right|^2\left|U_{\mu 3}\right|^2\sin\Delta_{13}\,, \tag{22b}$$

$$P_2(\mu,\mu) = -\left|U_{\mu 2}\right|^2\left(\left|U_{\mu 1}\right|^2 + \left|U_{\mu 3}\right|^2\cos\Delta_{13}\right)\,. \tag{22c}$$

The survival probability of ν_μ in the two-flavor framework is written as

$$P^{(2)}_{\nu_\mu\to\nu_\mu} = 1 - \sin^2 2\theta_{\mu\mu}\sin^2\frac{\Delta_{\mu\mu}}{2}\,, \tag{23}$$

where $\theta_{\mu\mu}$ is the mixing angle and $\Delta_{\mu\mu}$ is the mass-squared difference, which are obtained from the two-flavor analysis. We expect that all these parameters, especially $\Delta_{\mu\mu}$, to be shifted by the three-flavor effect. In order to estimate this effect, we rewrite them as

$$\sin^2 2\theta_{\mu\mu} = 1.0 - \varepsilon\,, \qquad \Delta_{\mu\mu} = \Delta_{13} + 2\delta\,, \tag{24}$$

where δ denotes the three-flavor effect, and ε stands for the difference from the maximal mixing. Both of them depend on the neutrino energy, base-line length, and the oscillation parameters. We assume that δ is smaller than Δ_{13} and $\Delta_{13} \sim O(1)$ for long base-line experiments.

From the atmospheric neutrino observations and K2K experiment, we already know that ν_μ oscillate to another flavor maximally at the first-dip, $\Delta_{\mu\mu}(E_\nu = E_{\rm dip}) = \pi$ for normal hierarchy. By using eq. (24), *i.e.*, $\Delta_{13} = \pi - 2\delta^{\rm dip}$, eq. (22) becomes

$$P_0(\mu,\mu) = 1 - 4\left|U_{\mu 3}^{\rm dip}\right|^2\left(1 - \left|U_{\mu 3}^{\rm dip}\right|^2\right)\cos^2\delta^{\rm dip}\,, \tag{25a}$$

$$P_1(\mu,\mu) = 2\left|U_{\mu 2}^{\rm dip}\right|^2\left|U_{\mu 3}^{\rm dip}\right|^2\sin 2\delta^{\rm dip}\,, \tag{25b}$$

$$P_2(\mu,\mu) = -\left|U_{\mu 2}^{\rm dip}\right|^2\left(\left|U_{\mu 1}^{\rm dip}\right|^2 - \left|U_{\mu 3}^{\rm dip}\right|^2\cos 2\delta^{\rm dip}\right)\,, \tag{25c}$$

and eq. (23)

$$P^{(2)}_{\nu_\mu\to\nu_\mu} = \varepsilon^{\rm dip}\,, \tag{26}$$

where the label "dip" in $|U_{\mu i}|$, δ, and ε means that these quantities take some fixed value at the first-dip energy $E_{\rm dip}$. From eqs. (25) and (26), we obtain

$$\delta^{\rm dip} \simeq \frac{4\left|U_{\mu 3}^{\rm dip}\right|^2\left(1 - \left|U_{\mu 3}^{\rm dip}\right|^2\right) - \left(1 - \varepsilon^{\rm dip}\right)}{4\Delta_{12}^{\rm dip}\left|U_{\mu 2}^{\rm dip}\right|^2\left|U_{\mu 3}^{\rm dip}\right|^2} + \frac{1 - \left|U_{\mu 2}^{\rm dip}\right|^2 - 2\left|U_{\mu 3}^{\rm dip}\right|^2}{4\left|U_{\mu 3}^{\rm dip}\right|^2}\Delta_{12}^{\rm dip}\,. \tag{27}$$

The first term of eq. (27) has to be zero because of the assumption $O(\delta) < 1$, and therefore we obtain

$$\left|U_{\mu 3}^{\rm dip}\right|^2 = \frac{\left(1 \pm \sqrt{\varepsilon^{\rm dip}}\right)}{2}\,. \tag{28}$$

When we take a negative sign in eq. (28), $\delta^{\rm dip}$ becomes

$$\delta^{\rm dip} \simeq -\frac{\Delta_{12}^{\rm dip}}{2}\left\{\left|U_{\mu 2}^{\rm dip}\right|^2 + \left(1 - \left|U_{\mu 2}^{\rm dip}\right|^2\right)\sqrt{\varepsilon^{\rm dip}} - \varepsilon^{\rm dip}\right\}\,. \tag{29}$$

Since the best fit value of the mixing angle is maximum from the experiments [1, 2], we take $\varepsilon^{\rm dip} = 0$. Thus, $\delta^{\rm dip}$ simplifies to

$$\delta^{\rm dip} \simeq -\frac{\Delta_{12}^{\rm dip}}{2}\left|U_{\mu 2}^{\rm dip}\right|^2\,. \tag{30}$$

From eq. (24), the larger mass-squared difference at $E_{\rm dip}$ is

$$\delta m_{13}^2(E_{\rm dip}) = \delta m_{\rm dip}^2 + \delta m_{12}^2(E_{\rm dip})\left|U_{\mu 2}^{\rm dip}\right|^2\,, \tag{31}$$

where

$$\delta m_{\rm dip}^2 \equiv \frac{2\pi E_{\rm dip}}{L}\,, \tag{32}$$

is from the two-flavor analysis. Equation (31) denotes that the order of magnitude of the three-flavor effect is roughly the same as that of the expected error of future experiments. Since we know that both δm_{12}^2 and $\left|U_{\mu 2}^{\rm dip}\right|^2$ are positive, the mass-squared difference from two-flavor analysis is slightly smaller than the larger mass-squared difference. We also obtain the relation between δ and the smaller mass-squared difference at the first peak $E_{\rm peak}$, where $\Delta_{\mu\mu}(E_{\rm peak}) = 2\pi$,

$$\delta^{\rm peak} \simeq -\frac{\Delta_{12}^{\rm peak}}{2}\left(1-\left|U_{\mu 2}^{\rm peak}\right|^2\right). \tag{33}$$

Thus, the larger mass-squared difference at $E_{\rm peak}$ is

$$\delta m_{13}^2(E_{\rm peak}) = \delta m_{\rm peak}^2 + \delta m_{12}^2(E_{\rm peak})\left(1-\left|U_{\mu 2}^{\rm peak}\right|^2\right), \tag{34}$$

where

$$\delta m_{\rm peak}^2 \equiv \frac{4\pi E_{\rm peak}}{L}, \tag{35}$$

is also from the two-flavor analysis. We obtain the same results for the inverted hierarchy. Since δm_{13}^2 is not changed by matter effect at E_ν <10 GeV, we obtain the relation between eq. (31) and eq. (34):

$$\delta m_{\rm dip}^2 + \delta m_{12}^2(E_{\rm dip})\left|U_{\mu 2}^{\rm dip}\right|^2 = \delta m_{\rm peak}^2 + \delta m_{12}^2(E_{\rm peak})\left(1-\left|U_{\mu 2}^{\rm peak}\right|^2\right). \tag{36}$$

By using the definition of $\delta m_{\rm dip,peak}^2$, we find

$$\frac{L}{2\pi}\frac{\delta m_{12}^2\left(E_{\rm dip}\right)\left|U_{\mu 2}^{\rm dip}\right|^2 - \delta m_{12}^2\left(E_{\rm peak}\right)\left(1-\left|U_{\mu 2}^{\rm peak}\right|^2\right)}{2E_{\rm peak}-E_{\rm dip}} = \pm 1, \tag{37}$$

where the sign of the right-hand side corresponds to the type of the mass hierarchy, the positive sign being that of the normal hierarchy. From the eqs. (31) and (34), we easily understand that the three-flavor effect for the larger mass-squared difference depends on the energy. When $\rho = 2.5({\rm g/cm^3})$ and $E_\nu \simeq O(1)$GeV, $\widetilde{\theta}_{12}$ is to shift away from θ_{12} towards 90°, which is obtained from eq. (19). From eqs. (4) and (5), the value of $|U_{\mu 2}|$ becomes 0. Because $\delta m_{12}^2(E_\nu)$ is also changed by the matter effect, which is shown in eq. (19), eq. (37) becomes

$$\frac{L}{2\pi}\frac{a(E_{\rm peak}) - \delta m_{12}^2\cos 2\theta_{12}}{E_{\rm dip}-2E_{\rm peak}} = \pm 1, \tag{38}$$

where the value of δm_{12}^2 and θ_{12} is that of vacuum one, which are listed in eq. (7). This relation suggest us that we can determine the sign of the δm_{13}^2 from the long base-line experiments, when we measure the energy of "peak" and "dip" precisely and we know the

value of the smaller mass-squared difference, mixing angle with small errors. This result cannot be obtained from the numerical analysis. This fact points out that we can pick up the three-flavor effect from the fitting function of the survival probability which is obtained from the experimental data.

4 transition probability

From eqs. (10) and (14), the transition probability, $\nu_\mu \to \nu_e$ is written as

$$P_{\nu_\mu \to \nu_e} = P_0(\mu, e) + P_1(\mu, e) \times \sin \Delta_{12} + P_2(\mu, e) \times 4 \sin^2 \frac{\Delta_{12}}{2}\,, \tag{39}$$

where

$$P_0(\mu, e) = 4 \left| U_{\mu 3} U_{e3} \right|^2 \sin^2 \frac{\Delta_{13}}{2}\,, \tag{40a}$$

$$P_1(\mu, e) = 2 \left\{ \mathrm{Re} \left(U_{e2}^* U_{\mu 2} U_{e3} U_{\mu 3}^* \right) \sin \Delta_{13} + 2 J_{\mathrm{MNS}}^{(\mu,e)} \sin^2 \frac{\Delta_{13}}{2} \right\}\,, \tag{40b}$$

$$P_2(\mu, e) = -\mathrm{Re} \left(U_{\mu 1} U_{e1}^* U_{e2} U_{\mu 2}^* + U_{\mu 2} U_{e2}^* U_{e3} U_{\mu 3}^* \cos \Delta_{13} \right) - J_{\mathrm{MNS}}^{(\mu,e)} \sin \Delta_{13}\,. \tag{40c}$$

Here, $J_{\mathrm{MNS}}^{(\mu,e)} = -A \sin \phi \equiv -J$. Under the two-flavor framework, this transition probability, $\nu_\mu \to \nu_e$ is written as

$$P_{\nu_\mu \to \nu_e}^{(2)} = \sin^2 \theta_{\mu e} \sin^2 \frac{\Delta_{\mu e}}{2}\,, \tag{41}$$

where $\Delta_{\mu e}$ is the mass-squared difference and $\theta_{\mu e}$ is the unknown mixing angle. We suppose that these two parameter, especially $\Delta_{\mu e}$, are changed by the three-flavor effect. As in the case of the survival probability, we rewrite these parameters as

$$\sin^2 \theta_{\mu e} = h\,, \qquad \Delta_{\mu e} = \Delta_{13} + 2\delta\,, \tag{42}$$

where δ is smaller than Δ_{13} and h, in general, can take an arbitrary value. Before estimating the three-flavor effect, let us calculate the value of h and δ for $\Delta_{12} = 0$. The transition probability becomes

$$P_{\nu_\mu \to \nu_e} = 4 \left| U_{\mu 3} U_{e3} \right|^2 \sin^2 \frac{\Delta_{13}}{2}\,, \tag{43}$$

with $\Delta_{12} = 0$. At the first peak of transition probability, $\Delta_{\mu e}(E_{\mathrm{peak}}) = \pi$, eqs. (41) and (43) become

$$\begin{aligned} P_{\nu_\mu \to \nu_e}^{(2)}(\Delta_{\mu e} = \pi) &= h\,, \\ P_{\nu_\mu \to \nu_e}(\Delta_{\mu e} = \pi) &= 4 \left| U_{\mu 3} U_{e3} \right|^2 \cos^2 \delta\,, \end{aligned} \tag{44}$$

respectively. From these equations, h and δ are solved as

$$
\begin{aligned}
h_0 &\equiv h(\Delta_{12} = 0, \Delta_{\mu e} = \pi) = 4\left|U_{\mu 3} U_{e3}\right|^2 , \\
\delta_0 &= \delta(\Delta_{12} = 0, \Delta_{\mu e} = \pi) = 0 ,
\end{aligned} \tag{45}
$$

when we suppose that $\Delta_{\mu e}$ is same as Δ_{13} without three-flavor effect.

When we assume that Δ_{12} is nonvanishing, but small, eq. (39) becomes

$$
P_{\nu_\mu \to \nu_e} = P_0(\mu, e) + P_1(\mu, e) \times \Delta_{12} + P_2(\mu, e) \times \Delta_{12}^2 + O(\Delta_{12}^3) , \tag{46}
$$

where $P_0(\mu, e)$, $P_1(\mu, e)$ and $P_2(\mu, e)$ are

$$
\begin{aligned}
P_0(\mu, e) &= 4\left|U_{\mu 3}\right|^2 |U_{e3}|^2 \cos^2\delta \\
&= 4\left|U_{\mu 3}\right|^2 |U_{e3}|^2 + O(\delta^2) , && \text{(47a)} \\
P_1(\mu, e) &= 2\mathrm{Re}\mathcal{A}_{23} \sin 2\delta - 4J\cos^2\delta \\
&= 4\delta \mathrm{Re}\mathcal{A}_{23} - 4J + O(\delta^2) , && \text{(47b)} \\
P_2(\mu, e) &= -\mathrm{Re}\mathcal{A}_{12} + \mathrm{Re}\mathcal{A}_{23} \cos 2\delta + J \sin 2\delta \\
&= -\mathrm{Re}\mathcal{A}_{12} + \mathrm{Re}\mathcal{A}_{23} + 2\delta J + O(\delta^2) , && \text{(47c)}
\end{aligned}
$$

at the first peak $E_{\rm peak}$. Here, the symbols $\mathcal{A}_{ij}$ are defined as

$$
\mathcal{A}_{ij} = U_{\mu i} U_{ei}^* U_{ej} U_{\mu j}^* . \tag{48}
$$

The value of U_{ij}, $\mathcal{A}_{ij}$, and δ is fixed at $E_{\rm peak}$*. We assume that h at $E_{\rm peak}$ is function of Δ_{12},

$$
h = h(\Delta_{12}) = h_0 + \sum_k a_k \Delta_{12}^k , \tag{49}
$$

where h_0 is equal to $4\left|U_{\mu 3} U_{e3}\right|^2$ and a_k are independent of the neutrino energy. From eqs. (47) and (49), δ can be solved as

$$
\delta \simeq \frac{(a_1 + 4J) + (\mathrm{Re}\mathcal{A}_{12} - \mathrm{Re}\mathcal{A}_{23} + a_2)\,\Delta_{12} + a_3 \Delta_{12}^2}{4\mathrm{Re}\mathcal{A}_{23} + 2J\Delta_{12}} . \tag{50}
$$

When we take the limit $\Delta_{12} \to 0$, δ must vanish because of eq. (45). Thus,

$$
a_1 = -4J , \tag{51}
$$

and

$$
\delta \simeq \frac{(\mathrm{Re}\mathcal{A}_{12} - \mathrm{Re}\mathcal{A}_{23} + a_2) + a_3\Delta_{12}}{4\mathrm{Re}\mathcal{A}_{23} + 2J\Delta_{12}} \Delta_{12} . \tag{52}
$$

*We drop here the label "peak" for $U^{\alpha j}$, $\mathcal{A}^{ij}$, and δ, for simplicity.

The denominator of eq. (52) becomes 0 under some conditions that are related to the value of the MNS matrix elements,

$$2\mathrm{Re}\hat{\mathcal{A}}_{23} + \hat{J}\hat{\Delta}_{12} = 0\,, \tag{53}$$

where $\hat{\mathcal{A}}_{23}$, $\hat{J}$, and $\hat{\Delta}_{12}$ denote some fixed value of them. By using the mixing angles θ_{ij} and CP phase ϕ, $\hat{\Delta}_{12}$ is written as

$$\hat{\Delta}_{12} = -\frac{2}{\sin\hat{\phi}}\left(\cos\hat{\phi} - \tan\hat{\theta}_{12}\tan\hat{\theta}_{23}\sin\hat{\theta}_{13}\right)\,. \tag{54}$$

Since δ does not diverge at the first peak, the numerator also has to be 0

$$\left(\mathrm{Re}\hat{\mathcal{A}}_{12} - \mathrm{Re}\hat{\mathcal{A}}_{23} + a_2\right) + a_3\hat{\Delta}_{12} = 0\,, \tag{55}$$

under the same condition. Using eq. (53), a_2 and a_3 become

$$a_2 = -\mathrm{Re}\hat{\mathcal{A}}_{12}\,, \quad \text{and} \quad a_3 = -\frac{1}{2}\hat{J}\,. \tag{56}$$

Finally, we obtain

$$\delta(E_{\mathrm{peak}}) \simeq -\frac{\Delta_{12}}{4}\frac{2\left(\mathrm{Re}\hat{\mathcal{A}}_{12} - \mathrm{Re}\mathcal{A}_{12} + \mathrm{Re}\mathcal{A}_{23}\right) - \hat{J}\Delta_{12}}{2\mathrm{Re}\mathcal{A}_{23} + J\Delta_{12}}\,, \tag{57}$$

and

$$h(E_{\mathrm{peak}}) = 4\left|U_{\mu 3}\right|^2 |U_{e3}|^2 - 4A\sin\phi\;\Delta_{12} - \mathrm{Re}\hat{\mathcal{A}}_{12}\Delta_{12}^2 + O(\Delta_{12}^3)\,. \tag{58}$$

Because the order of $\mathcal{A}_{ij}$ is less than 1, the shift of the first peak for the transition probability is not large. Equation (58) shows that the first-peak of the transition probability with $\delta_{\mathrm{MNS}} = 90^\circ$ is smaller than that with $\delta_{\mathrm{MNS}} = 270^\circ$. Since the value of $\mathrm{Re}\hat{\mathcal{A}}_{12}$ is negative for $\rho \simeq 3.0$ (g/cm^3), the first-peak of transition probability for the CP-conserved case is slightly larger than that with $\Delta_{12} = 0$. These features remain unchanged for the inverted hierarchy. But the shifting direction of the first-peak is different between the normal hierarchy and inverted one.

5 summary

In this article, we have estimated the three-flavor effect for the determination of the mass-squared difference and the mixing angle from the survival and transition probability of ν_μ. From both probabilities, the larger mass-squared difference is changed by the three-flavor effect. The order of magnitude of the difference between the larger mass-squared difference δm_{13}^2 and the mass-squared difference of the two-flavor analysis $\delta_{\mathrm{dip,peak}}$ is not only proportional to δm_{12}^2 but also to the MNS matrix elements. We also find the CP

phase dependence for the transition probability. If there is no three-flavor effect for ν_μ survival and transition probabilities, the first-dip energy of ν_μ survival probability and the first-peak energy of ν_μ transition one are as same as that from Δ_{13}. However, each of them is different from the value of Δ_{13}, which are shown from eq. (30) and eq. (57), because of three-flavor effect. This means that the value of δm^2_{13} from ν_μ survival probability is slightly different from that from $\nu_\mu \to \nu_e$ transition one. The order of them are not so smaller than that of the expected error for the δm^2_{13} in the future long base-line neutrino oscillation experiment. These results are useful to estimate the three-flavor effects for the value of the parameters which are obtained from the numerical analysis.

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